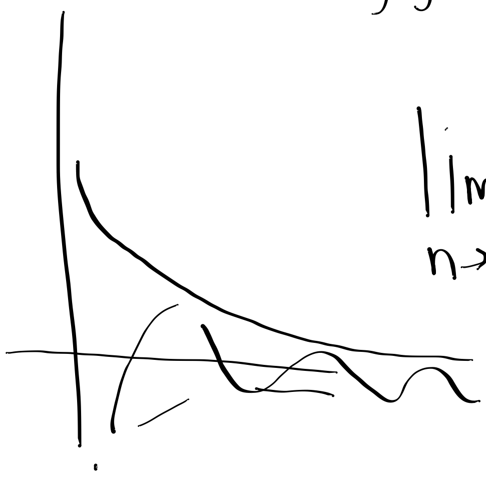
The geometric series $\sum_{n=1}^{\infty} ar^n$

converges to $\frac{ar}{1-r}$

iff $|r|<1$ [[diverges otherwise]]



$$\lim_{n\to\infty} \left\{ ar^n \right\}_{n=1}^{\infty} = 0$$

If $\sum_{n=1}^{\infty} a_n$ converges

then $\lim_{n\to\infty} \{a_n\}_{n=1}^{\infty} = 0$

converse fails [[harmonic series]]

If I have a series where $\{a_n\} \to 0$.
IDK about series...

Mar 1-7:36 AM

Harmonic Series

Music & mathematics

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots$

harmonics

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdots$

Michael Dillon

$$\sum_{n=1}^{\infty} \frac{1}{n} \geq 68$$

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$$\sum \frac{1}{n} \text{ ...... DIVERGES}$$

$$\underbrace{1+\frac{1}{2}}_{1.5}+\underbrace{\frac{1}{3}+\frac{1}{4}}_{>\frac{1}{4}+\frac{1}{4}=\frac{1}{2}}+\underbrace{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}}_{>\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}}+\cdots$$

Euler

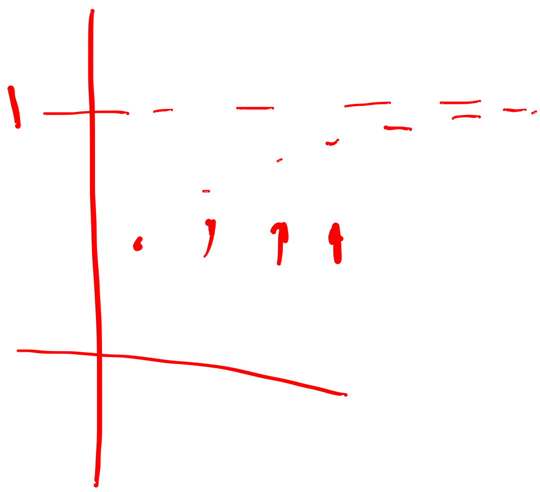
$$\sum_{p \text{ prime}} \frac{1}{p} \text{ diverges}$$

Test for Divergence

$$\lim_{n\to\infty} \{a_n\} > 0 \Rightarrow \sum a_n \text{ diverges}$$

$$\sum_{k=1}^{\infty} \frac{k}{k+1} \quad \text{divergs}$$

$$\left\{\frac{k}{k+1}\right\} \rightarrow 1$$



Series are, generally, very useful.

They allow us to model lots of f^n's we don't know Rules for . . .

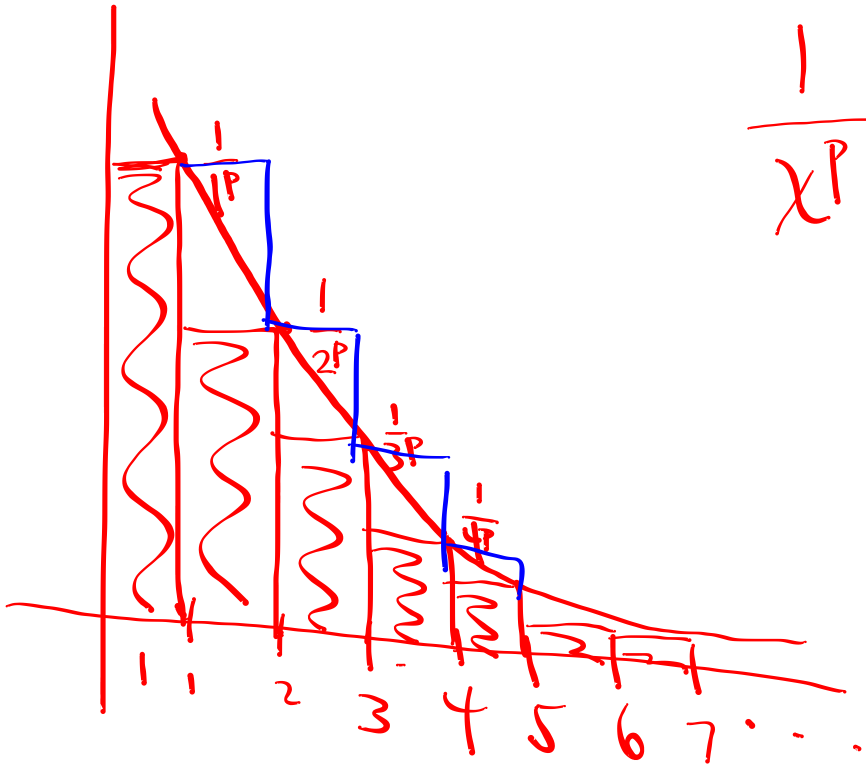
e.g. $\int e^{t^2} dt$

$$\sum_{k=1}^{\infty} \frac{1}{k^p}$$ "p - series"

$p = 1 \Rightarrow$ harmonic series (diverges)

$p = 0 \Rightarrow$ diverges

$p < 0 \Rightarrow$ diverges (test for divergence)



$\frac{1}{x^p}$

Series Integral test

Let $f(x)$ be a continuous f^n with $f(n) = a_n$.

Then $\int_1^\infty f(x)\,dx$ and $\sum a_n$

both converge OR
both diverge

Integral test & p-series

$p \neq 1$ $\quad \displaystyle\int_1^\infty \frac{1}{x^p}\,dx = \lim_{b\to\infty} \int_1^b x^{-p}\,dx$

$$= \lim_{b\to\infty} \left.\frac{x^{-p+1}}{1-p}\right|_1^b = \lim_{b\to\infty} \frac{1}{1-p}\left[\frac{1}{b^{p-1}} - 1\right]$$

when is $\displaystyle\lim_{b\to\infty} \frac{1}{b^{p-1}}$ existing?

$\searrow 0$

you want b^{p-1} to "stay" in the denominator

$p-1>0$

$p>1 \Rightarrow$ p series converge

$p<1 \Rightarrow$ p series diverge

$p=1 \Rightarrow$ p series diverge

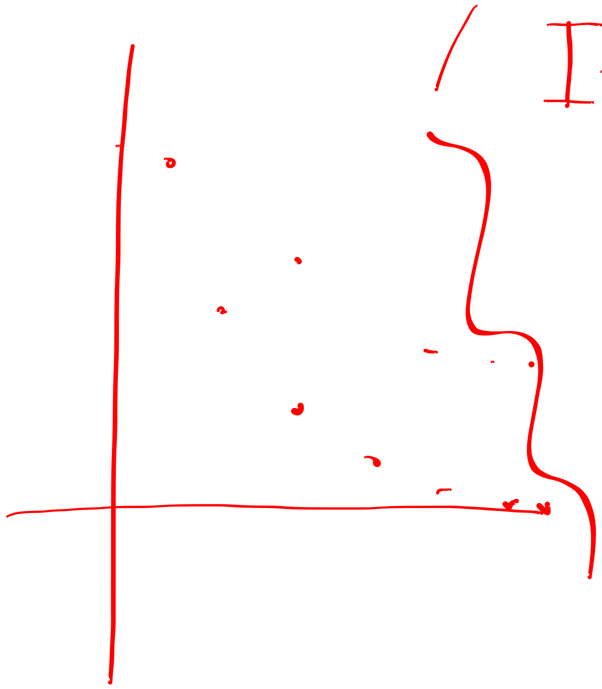
$p=-1 \Rightarrow$ DIVERGES BIG TIME

$p = -1$

$$\sum \frac{1}{k^{-1}} \Rightarrow \infty$$

A geometric series $\sum_{n=1}^{\infty} ar^n$ converges to $\frac{ar}{1-r}$

iff $|r| < 1$



If $\sum a_n$ convergs

then $\lim_{n\to\infty} a_n = 0$

Is there a sequence that $\to 0$ but $\sum a_n$ diverges?

Yes.... the harmonic series

$$\sum \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

$$\underbrace{1+\frac{1}{2}}_{1.5}+\underbrace{\frac{1}{3}+\frac{1}{4}}_{>\ \underbrace{\frac{1}{4}+\frac{1}{4}}_{\frac{1}{2}}}+\underbrace{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}}_{>\ \underbrace{\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}_{\frac{1}{2}}}+\cdots$$

Another look . . .

$$\sum_{n=1}^{\infty} a_n \text{ converges} \Rightarrow \lim_{n\to\infty} a_n = 0$$

contrapositive

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ diverges

Test for divergence

If a then b

If b false then a false.

$$\sum_{k=1}^{\infty} \frac{k}{k+1}$$

$$\frac{k(1)}{k\left(1+\frac{1}{k}\right)} \qquad \lim_{k\to\infty} \frac{k}{k+1} = 1 \neq 0$$

~~Geometric~~ Geometric Series

Harmonic Series

$\sum \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$

$$\sum_{k=1}^{\infty} \frac{1}{k^p} \longrightarrow$$

$p=1$ .... diverges
= harmonic

$p=0$ ..... $1+1+1\ldots$ diverges

$p<0$ ...... diverges..

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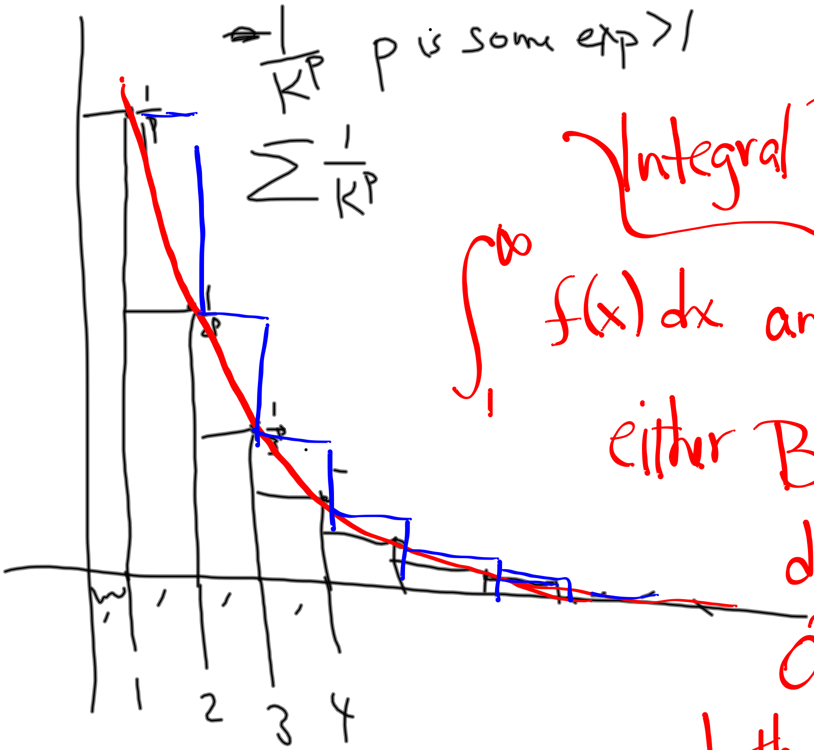
$\frac{1}{k^p}$ p is some exp > 1



$\sum \frac{1}{k^p}$

Integral Test for Series

$\int_1^\infty f(x)\,dx$ and $\sum f(n)$

either BOTH diverge

OR

both converge.

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$$\sum_{K=1}^{\infty} \frac{1}{K^p} \qquad \int_1^{\infty} \frac{1}{x^p}\,dx$$

$$= \lim_{b\to\infty} \int_1^b x^{-p}\,dx = \lim_{b\to\infty} \frac{x^{-p+1}}{-p+1}\Big|_1^b$$

$$= \lim_{b\to\infty} \frac{1}{1-p}\left(b^{-p+1} - 1^{-p+1}\right)$$

$$= \lim_{b\to\infty}\left(\frac{1}{1-p}\right)\left(\frac{1}{b^{p-1}} - 1\right)$$

$$\frac{1}{b^{p-1}} \xrightarrow{\lim} 0$$

$p-1>0$

$p>1$

$\int_1^{\infty} \frac{1}{x^p}\,dx$

& $\sum_{K=1}^{\infty} \frac{1}{K^p}$

Converge iff $p>1$

Geometric Series

Harmonic Series

p- Series